## [Representing Data]

(§9.4 worksheet I)

## (Confidence Intervals)

A survey reports that $62 \%$ of registered voters will vote in the next election with a $\pm 10 \%$ margin of error at the $95 \%$ confidence level.

Then the interval ( $52 \%, 72 \%$ ) (i.e. $52 \%-72 \%$ ) is a $95 \%$ confidence interval. This means we are $95 \%$ confident that between $52 \%$ and $72 \%$ of all registered voters will vote in the next election.

The confidence level of the interval expresses the confidence in the construction process of the interval. If new data was collected from a sample of the same size as used in the original sample, then a new interval would likely be constructed containing slightly different values. If this process for constructing intervals was repeated 100 times, then our $95 \%$ confidence level tells us that we'd expect 95 of those intervals to contain the true proportion of people who would vote in the next election.

## (example)

Suppose we are trying to find a confidence interval showing the mean weight of a certain breed of dog. We take a random sample of size 80 and construct a $95 \%$ confidence interval. If we repeated this 100 times with different samples we would get 100 different intervals. The $95 \%$ confidence level means we'd expect 95 of these intervals to contain the true mean weight of the dog breed. In some, the true mean might be in the middle, in others toward the high end in others toward the low end. We cannot make any prediction as to where in the interval the true mean may lie, just that we are $95 \%$ confident that the true mean is somewhere in the interval.


Facts:

- Random samples representative of the population of interest should be used in the construction of a confidence interval.
- The size of the confidence interval is affected by both the sample size and level of confidence.
- The larger the sample size, the shorter the interval (the more samples you use, the closer you are able to approximate the true values).
- The larger the confidence the wider the interval (the wider the interval, the more confidence you have that it contains the true value you are looking for).

1. Which sample would likely give us the best confidence interval (assume all samples are random and representative of the population)?
a. 50 voters from a population of 1500
b. 1000 voters from a population of 250,000
c. 1200 voters from a population of $1,000,000$

Explain your reasoning below.
2. A newspaper publishes a study indicating that $45 \%$ of the population likes spicy food with a margin of error of $\pm 7 \%$ at a confidence level of $90 \%$.
a. What is the confidence interval?
b. Would you conclude that most people like spicy food based on this interval?
c. What could you do to create an interval with less margin of error?
3. One study indicates that $50 \%$ of people are happy with the Affordable Care Act health insurance enrollment process with a margin of error of $\pm 5 \%$ at a $95 \%$ confidence level. A second, larger, survey indicated that $47 \%$ of people are happy with the Affordable Care Act health insurance enrollment process with a margin of error of $\pm 2 \%$ at a $95 \%$ confidence level. Is the percentage of the population who are happy with the Affordable Care Act health insurance enrollment process more likely closer to $50 \%$ or $47 \%$ ? Why?
4. One study indicates that $80 \%$ of adults know how to swim with a margin of error of $\pm 8 \%$ and a confidence level of 95\%. Another student claims they found that only $75 \%$ of adults know how to swim with a margin of error of $\pm 3 \%$ at a confidence level of $95 \%$. Which study used the larger sample? How can you tell?

Notes:

- Most studies use a $95 \%$ confidence level
- If you are constructing a 95\% confidence interval with a sample of size $n$ to estimate a proportion of a population/sample then the margin of error can be estimated to be $\sim \pm \frac{1}{\sqrt{n}}$
- If $p$ is the proportion of a sample, then the $95 \%$ confidence interval is

$$
\left(p-\frac{1}{\sqrt{n}}, p+\frac{1}{\sqrt{n}}\right)
$$

5. You are running for city mayor and conducted a survey of 120 people. Of those people 47 said they would vote for you.
a. If you construct a $95 \%$ confidence interval, what would be your margin of error?
b. What is your $95 \%$ confidence interval? Do you think you will win?
6. You sample 150 people and find that $48 \%$ of them exercise every morning.
a. If you construct a $95 \%$ confidence interval, what would be your margin of error?
b. What is your $95 \%$ confidence interval?
7. For $95 \%$ confidence, what reasonable sample size would guarantee that you have a margin of error less than $\pm 1 \%$ ?
8. For $95 \%$ confidence, what reasonable sample size would guarantee that you have a margin of error less than $\pm 3 \%$ ?
